of substance heating have been computed, knowledge of which plays an important part in the analysis of the sensitivity of solid HE to mechanical effects.

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WAVE PROPAGATION IN A UNIDIRECTIONAL COMPOSITE AS COMPARED WITH A LAMINAR ELASTIC SOLID

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UDC 539.3

In studying a unidirectional composite, the assumption is often used that the bonding fibers experience only tension–compression, and the binder only shear in areas parallel to the fibers. This hypothesis is based on purely qualitative considerations, and it is apparently impossible to give a sufficiently exact a priori estimate of the error it induces. Hence, the solution of test problems and a comparison of the results obtained with the solution for an elastic laminar medium are of interest. The propagation of stationary harmonic waves along fibers and the normal incidence of a plane stress wave on a half-space are examined in this paper as such problems. When the boundary load is a Heaviside function of the time, the second problem has been considered in [1] for an approximate model. Analysis of the solution obtained showed that it possesses all the fundamental singularities inherent in even more complex problems. At the same time, consideration of a plane wave is convenient for a numerical solution since it affords the possibility of being limited to the consideration of just two adjacent layers.

1. Let the composite consist of parallel fibers of thickness h with Young's modulus E and density ρ_1 embedded in one layer, between which the spaces are filled with layers of a binder of width H and shear modulus G and density ρ_2. We take the specimen thickness as unity, direct the y axis along the boundary between the fibers and the binder, and the x axis perpendicularly to the fibers (Fig. 1). In conformity with the model taken for the composite, the equations of motion of the components in the case when all the fibers move identically are of the form [1]

$$\frac{\partial^2 u}{\partial y^2} + \frac{2G}{Eh}\frac{\partial v}{\partial x}\Big|_{x=0} = \frac{1}{c_1^2}\frac{\partial^2 u}{\partial t^2}, \qquad (1.1)$$
$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{c_2^2}\frac{\partial^2 v}{\partial t^2}, \quad v|_{x=0} = v|_{x=H} = u,$$

where u and v are the displacements of the fibers and the binder, respectively, along the y axis, t is the time, $c_1 = \sqrt{E/\rho_1}$; $c_2 = \sqrt{G/\rho_2}$. The stresses are proportional to the corresponding strains $\sigma = E\partial u/dy$, $\tau = G\partial v/\partial x$.

Since the composite is an inhomogeneous body, it possess geometric dispersion which is manifest for harmonic waves as the frequency dependence of the phase velocity. Since the nonstationary waves can be repre-

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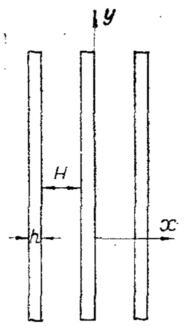

Fig. 1

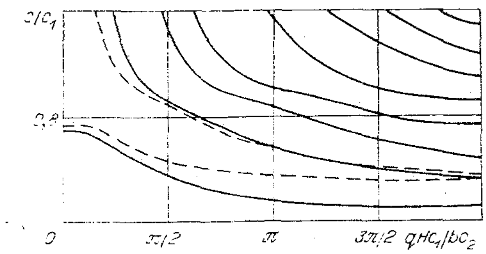

Fig. 2

sented in the form of a Fourier integral, i.e., expanded in harmonics, then certain preliminary qualitative information about its properties as compared to an elastic body can be extracted from an analysis of the dispersion curves of the approximate model.

We limit ourselves to a consideration of waves being propagated along the fibers. Let the composite occupy the whole plane and be subjected to harmonic oscillations

$$u(y,\ t) = u^0 e^{iq(y - ct)}, \tag{1.2}$$

where u^0 is the amplitude, c is the phase velocity, and q is the wave number. The representation (1.2) is equivalent to a Fourier transformation in y with the parameter q and in t with the parameter $-qc$. After such a transformation, the second of equations (1.1) is transformed into an ordinary differential equation with given boundary conditions. By solving it, v can be eliminated from the first equation and a dispersion relationship can be obtained for the approximate model

$$\frac{c_1^2 - c^2}{c} = \frac{2\rho_2 c_2}{q\rho_1 h}\,\frac{1 - \cos(qHc/c_2)}{\sin(qHc/c_2)}. \tag{1.3}$$

It can be seen that between every two of the hyperbolas $qc = \pi(2K+1)c_2/H$ ($k = 0, 1, 2, \ldots$) is a branch of the graph of the function $c = c(q)$, the dispersion curve $c = c_k(q)$, hence this graph has the form displayed in Fig. 2 (solid lines). Every dispersion curve corresponds to a certain mode of oscillations which differ by the distribution of the displacement v of the binder along x. The profile of v is found from the second equation in (1.1):

$$v(x,\ y,\ t) = \frac{u(y,\ t)}{\sin(qHc/c_2)}\left\{\sin\left[\frac{qHc}{c_2}\left(1 - \frac{x}{H}\right)\right] + \sin\left(\frac{qxc}{c_2}\right)\right\}.$$

The latter formula describes standing waves whose length diminishes for a given q as the mode number grows.

It follows from (1.3) that the velocity of infinitely short waves is zero for any mode, while the velocity of infinitely long waves tends to infinity for all modes except the zeroth. Letting q tend to zero, and assuming c bounded, we find this velocity for the lowest mode

$$c_\infty = \sqrt{Eh/(\rho_1 h + \rho_2 H)}.$$

The relationship obtained expresses the evident fact that the effective normal stiffness of the medium under consideration is proportional to Eh, while the mean density is $\rho_1 h + \rho_2 H$ ($(h + H)^{-1}$ is a proportionality factor).

In the dimensionless variables $\varkappa = qHc_1/bc_2$, $\zeta = c/c_1$, where $b = \rho_2 H/\rho_1 h$, the dispersion equation (1.3) has the form

$$(1 - \zeta^2)/\zeta = 2(1 - \cos(b\varkappa\zeta))/(\varkappa \sin(b\varkappa\zeta)).$$

Therefore, the form and location of the dispersion curves in a suitable coordinate system depend only on the dimensionless parameter b. As b grows, i.e., as the relative mass of the binder grows, the curves approach the coordinate axes.

Now, let us examine a medium consisting of alternating harder and more pliable elastic layers. For convenience later, we call the hard layers fiber, as before, and the pliable layers the binder.

The dispersion relationship for symmetric waves being propagated parallel to the layers in such a medium is given in [2, 3]. It contains five independent parameters. Again b can be selected as one of them, and the bonding factor $\psi = h/(h+H)$, the ratio between the Young's modulus of the fiber and the binder E/E_2, and their Poisson ratios ν_1 and ν_2, as the rest, say. The dashed lines in Fig. 2 correspond to the two lowest modes for

$b=1$, $\psi=0.5$, $E/E_2=10$, $\nu_1=\nu_2=0.25$ (the curves presented in the figure for the approximate model are constructed for the same values of the parameters).

The qualitative behavior of the curves is similar in both cases. Since the normal stiffness of the binder is taken into account in the exact theory, the mechanical system as a whole turns out to be stiffer also; consequently, the appropriate phase velocities for the lowest modes turned out to be higher for all q including small q. This is indeed valid for c_∞ (formula (36) in [3]). Since the case of the plane state of stress is considered, the Lamé constant λ in this formula should be replaced by $\lambda^*=2\lambda\mu/(\lambda+2\mu)$. The substantial quantitative discrepancy between the solid and dashed curves is observed only for sufficiently large q, but for waves whose length is on the order of or less than the characteristic dimension of the structural inhomogeneity of the medium, good agreement between the approximate model and the exact theory should not even be expected. Let us note that the velocity of infinitely short waves in a laminar elastic body equals the velocity of the shear wave in the binder c_2. They could be propagated at the velocity of Stonely waves ($<c_2$) but the latter are absent for the selected value of E/E_2 [4].

2. Let us turn to the problem of stress wave incidence on a half-space. We start with the approximate model. Let a stress

$$\sigma = E\left(1 - e^{-2tc_1/h}\right)\delta_0(t), \tag{2.1}$$

where δ_0 is the Heaviside unit function, be applied to the boundary of a composite occupying the half-space $y \geq 0$ and in a natural state at the initial instant. It is considered that the stress (2.1) is applied only to the fibers since the binder carries no normal load in conformity with the composite model taken (see [1] for more details).

The time dependence of σ in the form (2.1) is taken from the following considerations. The approximate model being examined is computed by studying the elastic field that occurs during a sudden discontinuity in the fibers [5-7]. It is customary to model this latter by an instantaneous drop in the stress to zero at the point of the discontinuity. In a supplementary problem obtained from the original by subtracting the initial field, this is equivalent to applying a load in the form of a Heaviside function. For such boundary conditions, an analytic solution (in finite form or in quadratures) is successfully constructed within the framework of the approximate model. However, the model should apparently not give results close to elasticity theory in this case since the contribution of the short waves, which the model describes poorly, to the solution can turn out to be quite substantial. Moreover, it is natural to consider the fiber discontinuity to occur in a certain finite time, for instance, $\sim h/c_1$. Consequently, the boundary load is selected continuous for $t=0$ and convergent to a constant in the time $\sim h/c_1$ in the test problem under consideration.

Equations (1.1) with the boundary condition (2.1) and zero initial conditions were solved by a finite-difference method. The difference scheme in the displacements, and the relationship of the spacings minimizing the numerical dispersion were taken according to the recommendations in [8]. The parameter denoted by $\Delta t/\alpha$ in [8] and characterizing the accuracy of the scheme was assumed to equal 0.05. Since the boundary load is continuous, the solution obtained turned out to be practically exact. A specific computation by the difference scheme was performed for $0 \leq y \leq y_0$, where the ordinate of the fictitious boundary y_0 was selected with a computation such that the wave would not succeed in reaching it in the time interval under consideration, and the boundary conditions on it would be assumed homogeneous. The domain of the solution was even bounded along x because of the periodicity of the problem. The middle line of the binder layer was taken as the fictitious boundary, and symmetry conditions were given on it. The stresses for the displacements found were determined by numerical differentiation. Results of the computations are presented below in a comparison with the solution for the elastic laminar medium.

The problem for the latter was formulated analogously: It was considered that the stress (2.1) was applied to all the fibers while the binder was load-free. The plane state of stress was examined. The domain of the computation was bounded by the middle lines of the fibers and the adjacent binder layers on which symmetry conditions were imposed, and by the fictitious boundary $y=y_0$ with the homogeneous boundary conditions. An explicit three-layer difference scheme for the solution of two-dimensional Lamé equations is obtained by a variational method. The second derivatives were approximated by ordinary central differences at the fiber and binder internal points (i.e., to the second order), while the difference relationships on the boundaries approximated the boundary conditions and the condition of rigid contact between the fiber and binder to the first order. The fiber was covered by a uniform mesh with spacing Δx, where the quantity of points over the half-thickness, including the boundary, was taken equal to six. The time spacing $\Delta t \leq \Delta x/\sqrt{a_1^2+a_2^2}$, where a_1 and a_2 are the respective velocities of the longitudinal and transverse waves in the fiber. The binder was also covered

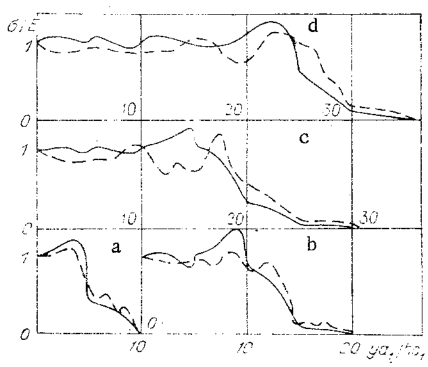

Fig. 3

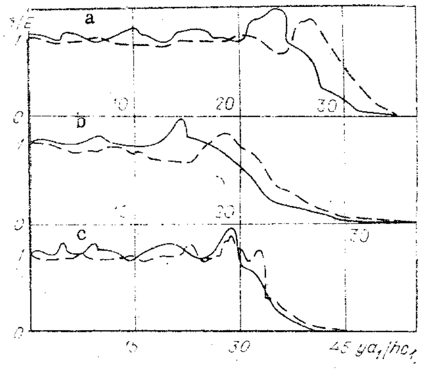

Fig. 4

by a uniform mesh, where Δy was taken the same as in the fiber, while Δx could be varied in such a way as b or ψ changed as to emplace all the necessary arrays in the operational storage of the electronic computer. The computation accuracy was checked by halving the difference scheme spacings for small values of the time.

The distribution averaged with respect to the fiber thickness, of the normal stress for the approximate model (solid lines) and the elastic body (dashes) along the coordinate y is presented in Fig. 3a-d ($t = 10$, 20, 30, 40 is, respectively, the time referred to h/a_1) for the same values of the parameters as in Fig. 2. Analogous graphs are given in Fig. 4 for other values of some of the parameters (a) $t = 35.4$, $b = 2$; b) $t = 37.5$, $\psi = 2$; c) $t = 60$, $\nu_1 = \nu_2 = 0$). It is seen that the results obtained within the framework of both models are in good qualitative agreement with each other, even for small values of the time. A certain lead in the maximum peak of the stress in the elastic medium as compared to the approximate model is explained by a somewhat lowered phase velocity, which the latter yields for long waves.

For small y the stress is practically stabilized in both cases, starting with a certain time. Here the approximate model exaggerates their value by 10% approximately, which is close to $E_2/(E + E_2)$. This is explained by the fact that the binder in an elastic medium takes on approximately that much of the load. If a boundary stress is applied even to the binder, then the graphs for small y come together substantially, but because of the low wave velocity in the binder this change in the boundary conditions does not at all influence their substantially nonstationary sections.

Therefore, the results obtained confirm the adequacy of the approximate composite model under consideration.

It would be interesting to compare the tangential stresses also on the boundary of the fiber and the binder, computed according to both methods, however, this cannot unfortunately be done successfully with sufficient accuracy. The fact is that these stresses are small quantities compared to the boundary load, which are commensurate with the error of the numerical computation in this case. For a more exact calculation of τ it is necessary to diminish the difference scheme spacings substantially, which is extremely complicated because of the limits to the electronic computer operational storage.

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ASYMPTOTIC ANALYSIS OF LONGITUDINAL AND BENDING WAVES PROPAGATED IN A SYSTEM OF TWO PLATES FASTENED AT AN ANGLE

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1. We introduce two coordinate systems so that in the left plate $x_1 \le 0$ while in the right $x_2 \ge 0$ (Fig. 1). The z_1 and z_2 axes are here directed normally to the plate surfaces so that the two half-planes ($z_1 = 0$ for $x_1 \le 0$ and $z_2 = 0$ for $x_2 \ge 0$) would coincide with their neutral planes. The y axis is along the line connecting the plates. Let an incident sinusoidal wave be propagated in the left plate. We examine the conditions for its passage through the boundary.

The plate vibrations are described by the following differential equations

$$D(\partial^4 w/\partial x^4 + 2\partial^4 w/\partial x^2 \partial y^2 + \partial^4 w/\partial y^4) + \rho h \partial^2 w/\partial t^2 = 0; \tag{1.1}$$

$$\frac{Eh}{1-\nu^2}\left(\frac{\partial^2 u}{\partial x^2} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial y^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial x \partial y}\right) = \rho h \frac{\partial^2 u}{\partial t^2}; \tag{1.2}$$

$$\frac{Eh}{1-\nu^2}\left(\frac{1-\nu}{2}\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{1+\nu}{2}\frac{\partial^2 u}{\partial x \partial y}\right) = \rho h \frac{\partial^2 v}{\partial t^2}, \tag{1.3}$$

where h is the thickness of the plates; E, elastic modulus; ν, Poisson ratio; and D, bending stiffness.

Displacements v_1, v_2 of points of the plate neutral planes along the y axis during vibrations u_1, u_2 along the x_1, x_2 axes will characterize the wave in the planes of the plates while the displacements w_1, w_2 along the z_1, x_2 axes, respectively, are bending waves [1, 2].

Solutions of the problem should satisfy eight boundary conditions on the hinge-supported edges

$$w = u = \partial v/\partial y = \partial^2 w/\partial y^2 = 0 \ (y = 0,\ l) \tag{1.4}$$

and eight juncture conditions on a common edge (x = 0)

$$u_1 = u_2 \cos\varphi + w_2 \sin\varphi; \tag{1.5}$$

$$w_1 = -u_2 \sin\varphi + w_2 \cos\varphi; \tag{1.6}$$

$$\partial w_1/\partial x_1 = \partial w_2/\partial x_2; \tag{1.7}$$

$$\sigma_{x_1} = \sigma_{x_2}\cos\varphi + R_{x_2}\sin\varphi; \tag{1.8}$$

$$R_{x_1} = -\sigma_{x_2}\sin\varphi + R_{x_2}\cos\varphi; \tag{1.9}$$

$$M_{x_1} = M_{x_2}; \tag{1.10}$$

$$v_1 = v_2; \tag{1.11}$$

$$\tau_{x_1 y} = \tau_{x_2 y}, \tag{1.12}$$

where $\varphi = \pi - \alpha$ (α is the angle between the plates), σ_{x_i} are the normal stresses, $\tau_{x_i y}$ are the shear stresses, M_{x_i} are bending moments relative to the x = 0 axis, $Q_{x_i} + \partial M_{x_i y}/\partial y$ (Q_{x_i} is the transverse force, $M_{x_i y}$ is the torque), i = 1, 2.

2. Let us first consider the particular case of the plane problem ($l = \infty$). Then v = 0, the solutions are independent of the variable y and the boundary conditions (1.4) drop out.

The juncture conditions on the common edge have the following form in this case

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